

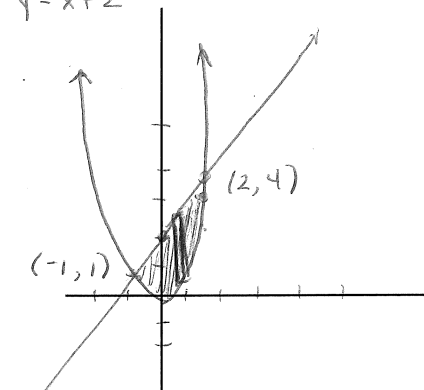
AP Calculus AB
Area Between Curves Worksheet #3

Name Key

Sketch the graphs, shade the bounded region and find the area bounded by the given expressions.

1) $x = y - 2$ and $y = x^2$

$y = x + 2$



$$\int_{-1}^2 (x+2-x^2) dx$$

$$\left. \frac{1}{2}x^2 + 2x - \frac{1}{3}x^3 \right|_{-1}^2$$

$x^2 = x + 2$

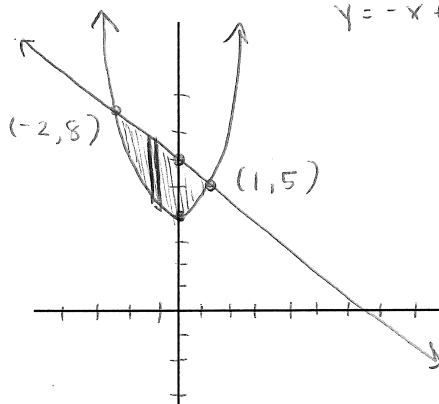
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ $x = -1$

2) $y = x^2 + 4$ and $x + y = 6$

$y = -x + 6$



$$\int_{-2}^1 (-x+6-(x^2+4)) dx$$

$$\int_{-2}^1 (-x^2 - x + 2) dx$$

$$\left. -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x \right|_{-2}^1$$

$x^2 + 4 = -x + 6$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

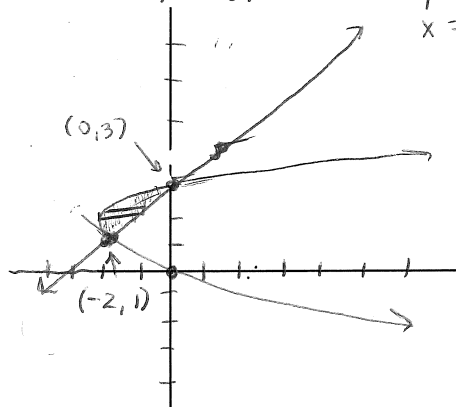
$x = -2$ $x = 1$

3) $x = y^2 - 3y$ and $x - y + 3 = 0$

$y(y-3)$

$y = x + 3$

$x = y - 3$



$$\int_1^3 (y-3-(y^2-3y)) dy$$

$$\int_1^3 (-y^2 + 4y - 3) dy$$

$$\left. -\frac{1}{3}y^3 + 2y^2 - 3y \right|_1^3$$

$y^2 - 3y = y - 3$

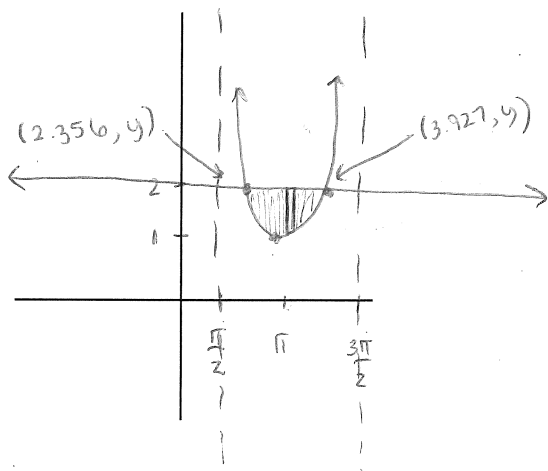
$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y = 3$ $y = 1$

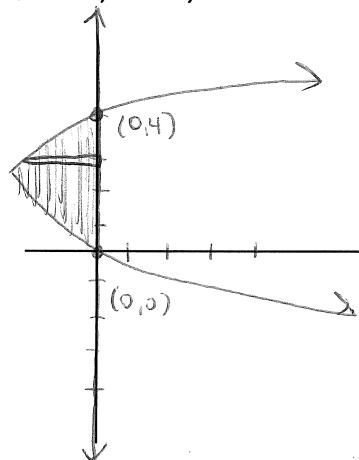
calc:

4) $y = \sec^2 x$, $y = 2$, $x = \pi/2$, and $x = 3\pi/2$



$$\int_{2.356}^{3.927} 2 - \sec^2 x \, dx$$

5) $x = y^2 - 4y$ and $x = 0$



$$\int_0^4 0 - (y^2 - 4y) \, dy$$

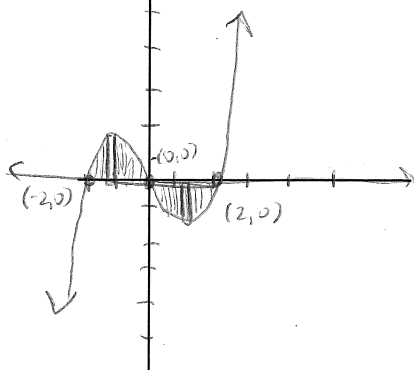
$$\int_0^4 -y^2 + 4y \, dy$$

$$-\frac{1}{3}y^3 + 2y^2 \Big|_0^4$$

$$\begin{aligned} y^2 - 4y &= 0 \\ y(y - 4) &= 0 \\ y &= 0 \quad y = 4 \end{aligned}$$

6) $y = x^3 - 4x$ and $y = 0$

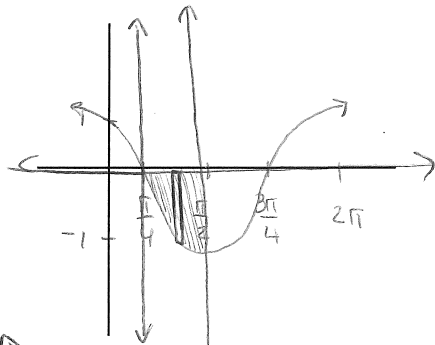
$$\begin{aligned} x(x^2 - 4) \\ x(x+2)(x-2) \\ x=0 \quad x=-2 \quad x=2 \end{aligned}$$



$$\int_{-2}^0 x^3 - 4x - 0 \, dx + \int_0^2 0 - (x^3 - 4x) \, dx$$

$$\frac{1}{4}x^4 - 2x^2 \Big|_{-2}^0 + \frac{1}{4}x^4 + 2x^2 \Big|_0^2$$

7) $y = \cos(2x)$, $x = \pi/4$, and $x = \pi/2$ $y = 0$

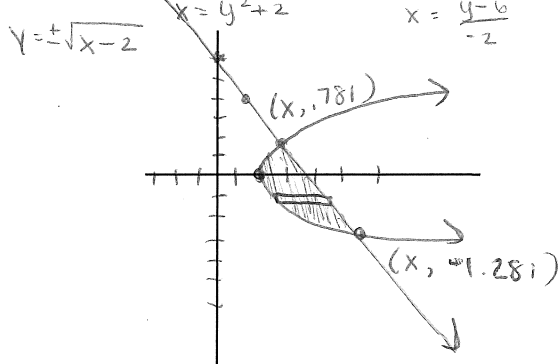


$$\int_{\pi/4}^{\pi/2} 0 - \cos(2x) dx = -\frac{1}{2} \sin 2x \Big|_{\pi/4}^{\pi/2}$$

$$\int_{\pi/4}^{\pi/2} -\cos 2x dx$$

calc:

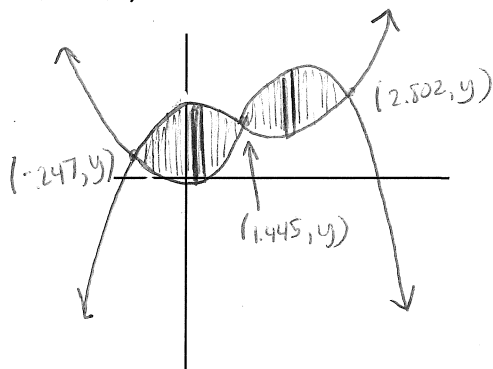
8) $y^2 = x - 2$, $y = -2x + 6$



$$\int_{-1.281}^{7.81} \frac{6-y}{2} - (y^2+2) dy$$

calc:

9) $y = x^3 - 4x^2 + 3x + 3$ and $y = -x^3 + 4x^2 - 3x + 1$

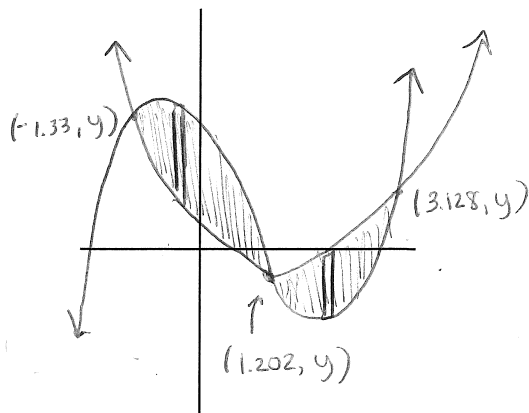


$$\int_{-2.47}^{1.445} (x^3 - 4x^2 + 3x + 3) - (-x^3 + 4x^2 - 3x + 1) dx$$

$$+ \int_{1.445}^{2.802} (-x^3 + 4x^2 - 3x + 1) - (x^3 - 4x^2 + 3x + 3) dx$$

calc:

10) $y = x^3 - 2x^2 - 5x + 6$ and $y = x^2 - 3x + 1$



$$\int_{-1.33}^{1.202} (x^3 - 2x^2 - 5x + 6) - (x^2 - 3x + 1) dx$$

$$+ \int_{1.202}^{3.128} (x^2 - 3x + 1) - (x^3 - 2x^2 - 5x + 6) dx$$

